

# Decoherence within a single atom

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An "almost diagonal" reduced density matrix (in coordinate representation) is usually a result of environment induced decoherence and is considered the sign of classical behavior. We point out that the proton of a ground state hydrogen atom can possess such a density matrix. This example demonstrates that the "almost diagonal" structure may derive from an interaction with a low number of degrees of freedom which play the role of the environment. We also show that decoherence effects in our example can only be observed if the interaction with the measuring device is significantly faster than the interaction with the environment (the electron). In the opposite case, when the interaction with the environment is significant during the measurement process, coherence is maintained. Finally, we propose a neutron scattering experiment on cold He atoms to observe decoherence which shows up as an additional positive contribution to the differential scattering cross section. This contribution is inversely proportional to the bombarding energy.

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Decoherence [1] is a remarkable quantum phenomenon which plays a central role in the (not yet fully understood) emergence of classical properties. It is also a practically important effect in quantum optics, atomic physics and mesoscopic systems. Therefore, it can be of interest to discuss some of its less known aspects. In the present paper we show on a simple example that contrary to the common belief,

1. an "almost diagonal" (in coordinate representation) density matrix may be the result of an interaction with only a few degrees of freedom
2. an "almost diagonal" density matrix does not necessarily imply the loss of coherence
3. interaction may maintain rather than destroy coherence.

We also discuss the conditions when and how decoherence can be observed.

Our primary example is a single ground state hydrogen atom. Later we shall also consider the case of a helium atom.

The wave function of a freely moving hydrogen atom in the ground state may be written in coordinate representation in the product form

$$\Phi(\vec{r}_p, \vec{r}_e) = \psi(\vec{R})\varphi_0(\vec{r}) \quad (1)$$

where

$$\vec{R} = \frac{m_p \vec{r}_p + m_e \vec{r}_e}{m_p + m_e} \quad (2)$$

stands for the coordinates of the center of mass and

$$\vec{r} = \vec{r}_e - \vec{r}_p \quad (3)$$

for the relative coordinates. The vectors  $\vec{r}_p$ ,  $\vec{r}_e$  refer to the position of the proton and that of the electron, respectively, while  $m_p$  and  $m_e$  are the corresponding masses. In Eq.(1)

$$\varphi_0(\vec{r}) = \frac{1}{\sqrt{\pi a_B^3}} \exp\left(-\frac{r}{a_B}\right) \quad (4)$$

is the ground state of the hydrogen atom, where  $a_B = \frac{\hbar^2}{m_e e^2} = 0.529 \times 10^{-8} \text{cm}$  stands for the Bohr radius. As the Schrödinger equation of the hydrogen atom separates in the  $\vec{R}$ ,  $\vec{r}$  coordinates, the time evolution of the center of mass wave function does not influence the state  $\varphi_0(\vec{r})$ , and the product form (1) persists. Nevertheless, when expressing the total wave function in the original coordinates  $\vec{r}_p$ ,  $\vec{r}_e$ , the state of the hydrogen atom is not of a product form, instead, it is an entangled state. Indeed, both  $\psi(\vec{R})$  and  $\varphi_0(\vec{r})$  depend on both  $\vec{r}_p$  and  $\vec{r}_e$ . For simplicity suppose that  $\Delta P a_B / \hbar \ll m_p / m_e$  ( $\Delta P$  standing for the momentum uncertainty of the center of mass), or, equivalently, that

$$\Delta V \ll \frac{\hbar}{m_e a_B} \approx v_e \approx 2 \times 10^6 \frac{m}{s} \quad (5)$$

( $\Delta V$  being the velocity uncertainty of the center of mass and  $v_e$  standing for the velocity of the electron), then  $\psi(\vec{R}) \approx \psi(\vec{r}_p)$ , and we arrive at the Born-Oppenheimer approximation [2]

$$\Phi(\vec{r}_p, \vec{r}_e) = \psi(\vec{r}_p)\varphi_0(\vec{r}_e - \vec{r}_p) \quad (6)$$

Entanglement implies that the proton is not in a pure state, i.e., its density matrix  $\rho(\vec{r}_p, \vec{r}_p')$  is not a single projector. Indeed, using expression (6) we get

$$\rho(\vec{r}_p, \vec{r}_p') = \int d^3\vec{r}_e \Phi(\vec{r}_p, \vec{r}_e) \Phi^*(\vec{r}_p', \vec{r}_e) \quad (7)$$

$$= \psi(\vec{r}_p) \psi^*(\vec{r}_p') \left(1 + s + \frac{1}{3}s^2\right) e^{-s} \quad (8)$$

where  $s = |\vec{r}_p - \vec{r}_p'|/a_B$ . Provided that the width of the center of mass wave function is much larger than the Bohr radius, the reduced density matrix (8) is "almost diagonal", i.e., its width (which is approximately  $2 a_B$ ) is much smaller than its length along the diagonal. The condition for this is  $\Delta P a_B/\hbar \ll 1$ , or

$$\Delta V \ll \frac{\hbar}{m_p a_B} \approx 10^3 \frac{m}{s}. \quad (9)$$

It is a much stronger condition than that of the Born-Oppenheimer approximation (cf. Eq.(5)). Let us emphasize that the "almost diagonal" structure in this example is due to the interaction with a single electron.

An "almost diagonal" density matrix (in coordinate representation) is usually a result of "environment induced decoherence" and is considered the sign of classical behavior. In the present paper we consider the validity of this expectation and discuss the physical meaning of density matrices like (8).

It is of worth mentioning that we do not need to rely upon the Born-Oppenheimer approximation in order to arrive at an "almost diagonal" density matrix. That approximation is only convenient because it allows one to perform the integration in Eq.(7) explicitly, independently of the actual form of the center of mass wave function. More generally, Eqs.(1), (7) imply

$$|\rho_p(\vec{r}_p, \vec{r}_p')| \leq \left(\max_{\vec{R}} |\psi(\vec{R})|^2\right) \left(1 + s + \frac{1}{3}s^2\right) e^{-s} \quad (10)$$

This inequality shows that the offdiagonal matrix elements of the reduced density matrix decay at least exponentially when increasing  $s = |\vec{r}_p - \vec{r}_p'|/a_B$ .

As an example, suppose that

$$\begin{aligned} \psi(\vec{R}) = & \frac{1}{(2\pi\delta^2)^{\frac{3}{4}}} \frac{1}{\left(1 + i\frac{\hbar t}{2M\delta^2}\right)^{\frac{3}{2}}} \exp\left(-i\frac{P_0^2 t}{2M\hbar}\right. \\ & \left. - \frac{(\vec{R} - \vec{R}_0 - \frac{\vec{P}_0}{M}t)^2}{4\delta^2\left(1 + i\frac{\hbar t}{2M\delta^2}\right)} + i\frac{\vec{P}_0 \cdot (\vec{R} - \vec{R}_0)}{\hbar}\right) \end{aligned} \quad (11)$$

where  $M = m_p + m_e \approx m_p$ . Eq. (11) describes a Gaussian wave packet moving in free space. If condition (5) does not hold, the integral in Eq.(7) cannot be expressed in a closed analytical form. Assuming the validity of (5), however, we obtain Eq.(8). We get a narrow density matrix (an "almost diagonal" one) if

$$\Delta x = \sqrt{\delta^2 + \left(\frac{\hbar t}{2M\delta}\right)^2} \gg a_B.$$

The deviation of the reduced density matrix of the proton from the pure state can be characterized by  $\text{Tr}\hat{\rho}_p^2$ . It

is unity for a pure state and less than unity (but positive) for a mixed state. Using Eqs.(11), (5) we have

$$\begin{aligned} \text{Tr}\hat{\rho}_p^2 = & \frac{z^3}{2\sqrt{\pi}} \int_0^\infty ds s^2 \left(1 + s + \frac{1}{3}s^2\right)^2 \\ & \times \exp\left(-2s - \frac{s^2 z^2}{4}\right) \end{aligned} \quad (12)$$

where

$$z = \frac{a_B}{\Delta x} = \frac{a_B}{\sqrt{\delta^2 + \left(\frac{\hbar t}{2M\delta}\right)^2}} \quad (13)$$

It is easy to see that in case of an "almost diagonal" density matrix (i.e., when  $z \ll 1$ )  $\text{Tr}\hat{\rho}_p^2$  is much smaller than one (in fact, it goes to zero with  $z$  as  $z^3$ ), while in the other extreme of very large  $z$  it approaches unity, which corresponds to a pure state.

As it is well known, a reduced density matrix gives all the possible information about the results of measurements done on the corresponding subsystem. Let us emphasize that such measurements are assumed to be infinitely fast, so that they give a snapshot about the state of the subsystem. It is obvious that coordinate measurements do not reveal the structure of the density matrix, as the probability distribution of the coordinates are given by the diagonal elements  $\rho_p(\vec{r}_p, \vec{r}_p)$  which is independent of the decay of the offdiagonal elements. Note that in the Born-Oppenheimer approximation (cf. Eq.(6))  $\rho_p(\vec{r}_p, \vec{r}_p) = |\psi(\vec{r}_p)|^2$ , which is completely independent of the interaction between the proton and the electron. In contrast, momentum measurements are sensitive for the behavior of the offdiagonal elements of the reduced density matrix in coordinate representation. The probability distribution of the momentum is determined by the diagonal elements of the reduced density matrix in momentum representation, i.e., by

$$\begin{aligned} \tilde{\rho}_p(\vec{p}_p, \vec{p}_p) = & \frac{1}{(2\pi\hbar)^3} \int d^3\vec{r}_p \int d^3\vec{r}_p' \rho_p(\vec{r}_p, \vec{r}_p') \\ & \times \exp\left(-\frac{i}{\hbar}\vec{p}_p \cdot (\vec{r}_p - \vec{r}_p')\right) \end{aligned} \quad (14)$$

Introducing coordinates parallel and perpendicular to the diagonal of the reduced density matrix (in coordinate representation), i.e.,

$$\vec{r}_{\parallel} = \frac{1}{2}(\vec{r}_p + \vec{r}_p') \quad (15)$$

$$\vec{r}_{\perp} = \vec{r}_p - \vec{r}_p' \quad (16)$$

we may write

$$\begin{aligned} \tilde{\rho}_p(\vec{p}_p, \vec{p}_p) = & \frac{1}{(2\pi\hbar)^3} \int d^3\vec{r}_{\perp} \exp\left(-\frac{i}{\hbar}\vec{p}_p \cdot \vec{r}_{\perp}\right) \\ & \times \int d^3\vec{r}_{\parallel} \rho_p(\vec{r}_{\parallel} + \frac{1}{2}\vec{r}_{\perp}, \vec{r}_{\parallel} - \frac{1}{2}\vec{r}_{\perp}) \end{aligned} \quad (17)$$

This last expression shows that the momentum distribution is proportional to the Fourier transform of the off-diagonal elements of the density matrix in coordinate representation, after having averaged them along the diagonal. Obviously, the narrower the density matrix (in coordinate representation) becomes, the broader the momentum distribution will be. Using the Born-Oppenheimer approximation (6) and the explicit expression (11), we get

$$\tilde{\rho}_p(\vec{p}_p, \vec{p}_p) = \frac{a_B^3}{2\pi^2\hbar^3} \frac{1}{q} \int_0^\infty ds s \sin(qs) \times \left(1 + s + \frac{1}{3}s^2\right) \exp\left(-s - \frac{s^2 z_0^2}{8}\right) \quad (18)$$

where  $q = |\vec{p}_p - \vec{P}_0|a_B/\hbar$  and  $z_0 = a_B/\delta$ . Note that the momentum distribution is independent of the time, which is a natural consequence of the fact that momentum (unlike coordinates) is now a conserved quantity.

It is instructive to evaluate expression (18) in two extreme situations: for  $a_B \gg \delta$  and  $a_B \ll \delta$ . The former case corresponds to a pure state of the proton, while in the latter case one has an "almost diagonal" mixed state. For  $a_B \gg \delta$  we get

$$\tilde{\rho}_p(\vec{p}_p, \vec{p}_p) = \left(\frac{2}{\pi}\right)^{\frac{3}{2}} \frac{\delta^3}{\hbar^3} \exp\left(-2 \frac{(\vec{p} - \vec{P}_0)^2 \delta^2}{\hbar^2}\right) \quad (19)$$

It coincides with the momentum distribution of the center of mass, and is independent of the electronic motion. In the opposite case (which is our case of interest), for  $a_B \ll \delta$  we have

$$\tilde{\rho}_p(\vec{p}_p, \vec{p}_p) = \frac{8a_B^3}{\pi^2\hbar^3} \frac{1}{(1+q^2)^4} \quad (20)$$

This coincides with the momentum distribution of the atomic electron (if  $\vec{q}$  is identified with the momentum of the electron times  $a_B/\hbar$ ) and is independent of the center of mass motion. Let us emphasize that it is the momentum distribution of the proton, and the above coincidence is a consequence of momentum conservation and that the variance of the total momentum is negligible. Obviously, the width of distribution (20) is  $\approx \hbar/a_B$ , much larger than  $\hbar/\delta$ , the momentum uncertainty corresponding to the state (11). This broadening marks the "almost diagonal" structure of  $\rho(\vec{r}_p, \vec{r}_p')$ . Below we shall point out that this sign of decoherence also shows up in neutron scattering cross sections which are experimentally more accessible.

The above discussion raises the obvious question how one can then observe diffraction with atomic beams [3], why interference is not destroyed by decoherence. In a typical diffraction experiment done with atoms the condition of "almost diagonal" structure of  $\rho(\vec{r}_p, \vec{r}_p')$ , Eq.(9) is indeed fulfilled. The essential point of the explanation is

that the density matrix which characterizes a subsystem (in our case the proton) refers to instantaneous (infinitely fast) measurements. This means that one can observe decoherence effects in a measurement done on the proton if during the interaction between the proton and the measuring device the state of the electron does not change significantly. As an estimate, one may compare the momentum actually gained by the electron from the proton during the measurement to the momentum change of the electron that would take place if the electron followed the proton instantaneously (as in the Born-Oppenheimer approximation). One can also tell that decoherence effects become observable if the measurement is so fast that the Born-Oppenheimer approximation for the electronic motion fails. Later we shall express the condition in a concrete situation quantitatively. The condition of observing decoherence is obviously not fulfilled in a diffraction experiment: the time of the measurement here is the time of flight between the grid and the screen. During this time the interaction between the proton and the electron maintains the form (1) of the state of the atom, i.e., the Born-Oppenheimer approximation is valid all the time. For definiteness, let us consider a two slit experiment where the distance between the slits is much larger than  $a_B$  but smaller than  $\delta$ . Be  $\alpha(\vec{R}, t)$  and  $\beta(\vec{R}, t)$  outgoing waves emerging from the first and the second slit, respectively. Then the center of mass wave function can be written as

$$\psi(\vec{R}, t) = a\alpha(\vec{R}, t) + b\beta(\vec{R}, t), \quad (21)$$

where  $a$  and  $b$  are the probability amplitudes that the atom goes through the first and the second slit, respectively. Eq.(1) remains valid until the atom reaches the screen, thus Eq.(21) is also valid during the time of flight. Initially, at the instant of time when the atom has passed through the slits  $\alpha(\vec{R}, t)$  and  $\beta(\vec{R}, t)$  are still narrow, separated wave packets, but later on they broaden and overlap. At the (only approximately determined) time instant  $t_0$  when the atom hits the screen Eqs.(1)-(8) are still valid, thus we get for the probability distribution  $P(\vec{r}_p)$  of finding the proton near a given point  $\vec{r}_p$  the expression

$$P(\vec{r}_p) = |a\alpha(\vec{r}_p, t_0) + b\beta(\vec{r}_p, t_0)|^2. \quad (22)$$

This includes interference terms, too. Thus we see that despite of the "almost diagonal" density matrix (which has had such form already initially) coherence is not destroyed. Paradoxically, this is due to the same interaction (Coulomb attraction between the electron and the proton) which is responsible for the "almost diagonal" structure of the density matrix.

In order to shed more light on the situation, let us consider what we would get if the interaction between the electron and the proton were turned off when the atom had passed through the slits. This can be done

in principle even experimentally by making use of the fact that during fast electronic processes the nucleus remains “frozen”. In molecular physics this is called the Franck-Condon principle [4]. If the atom is irradiated by a suitable ultraviolet laser beam, it can be ionized. This is a fast process which practically does not influence the position and motion of the proton, but eliminates the interaction between the proton and the electron. In order to see the consequences let us express the initial state  $\Phi(\vec{r}_p, \vec{r}_e, t = 0)$  (i.e., still before the ionization) in the Schmidt (or biorthogonal) representation [5], i.e., as

$$\begin{aligned}\Phi(\vec{r}_p, \vec{r}_e, t = 0) &= \sum_j c_j \chi_j(\vec{r}_p) \xi_j(\vec{r}_e) \\ &\approx a \alpha(\vec{r}_p, t = 0) \varphi_0(\vec{r}_e - \vec{r}_{s1}) \\ &\quad + b \beta(\vec{r}_p, t = 0) \varphi_0(\vec{r}_e - \vec{r}_{s2}).\end{aligned}\quad (23)$$

Here  $\vec{r}_{s1}$ ,  $\vec{r}_{s2}$  stand for the slit positions,  $\chi_j(\vec{r}_p)$ -s (i.e.,  $\alpha(\vec{r}_p, t = 0)$  and  $\beta(\vec{r}_p, t = 0)$ ) are the eigenstates of the reduced density matrix of the proton, while  $\xi_j(\vec{r}_e)$ -s (i.e.,  $\varphi_0(\vec{r}_e - \vec{r}_{s1})$  and  $\varphi_0(\vec{r}_e - \vec{r}_{s2})$ ) are the eigenstates of the reduced density matrix of the electron. The approximate orthogonality of  $\varphi_0(\vec{r}_e - \vec{r}_{s1})$  and  $\varphi_0(\vec{r}_e - \vec{r}_{s2})$  follows because  $|\vec{r}_{s1} - \vec{r}_{s2}| \gg a_B$ . During and after the ionization process the states of the electron-photon system evolve in time unitarily. The states  $\chi_j(\vec{r}_p)$  of the proton have a separate unitary time evolution. Therefore, we have

$$\begin{aligned}P(\vec{r}_p) &= \sum_j |c_j|^2 \left| \left( \hat{U}_{t_0} \varphi_j \right) (\vec{r}_p) \right|^2 \\ &\approx |a|^2 |\alpha(\vec{r}_p, t_0)|^2 + |b|^2 |\beta(\vec{r}_p, t_0)|^2,\end{aligned}\quad (24)$$

i.e., interference terms are absent. It can also be understood as a consequence of the fact that the “which way” information after the ionization can be obtained by measuring the electron without disturbing the proton.

In the presence of the interaction, however, interference and coherence is restored. In that case, of course, separate unitary time evolutions for the proton and the electron do not exist, instead, they move together and the atom behaves (in free space or in slowly varying potentials) as a single unit.

Let us return now to the question how one could observe experimentally the consequences of the “almost diagonal” density matrix (8). As mentioned above, decoherence effects emerge if during the measurement done on the proton the interaction between the electron and the proton is negligible. One possibility, namely, photoionization behind the slit has already been mentioned. In this case the duration of the measurement is unchanged, but the electron-proton interaction is “turned off”. Another possibility is to perform a very fast measurement on the proton. Below we suggest such an experiment.

Let us consider low energy (a few eV-s) neutron scattering at a helium atom, prepared in a state where the width of the center of mass wave function is much larger

than the atomic size. Low energy neutrons interact with the nucleus through a contact potential

$$g \delta(\vec{r}_n - \vec{r}_\alpha) \quad (25)$$

where  $g = 2\pi\hbar^2 a/\mu$ ,  $a$  standing for the scattering length and  $\mu$  being the reduced mass of the neutron-nucleus system. There is also an interaction between the magnetic moments of the neutron and the electron, however, in case of the helium atom the contributions of the two electrons to the magnetic scattering cancel each other, provided that an inelastic scattering (excitation of the electrons to higher levels) is energetically not possible. This is why we need helium for this experiment rather than hydrogen.

The condition for observing decoherence is that during the interaction time  $d/v$  ( $d$  standing for the nucleus size and  $v$  for the neutron velocity) the momentum transferred to the electron

$$\approx \frac{d}{v} \frac{1}{4\pi\epsilon_0} \frac{2q^2}{a_B^2} \approx \frac{m_e v_e^2}{v} \frac{d}{a_B}$$

is much less than the total change of the momentum of the electrons well after the collision,  $2m_e v$ . (As before,  $m_e$  and  $v_e$  stand for the electron mass and velocity, respectively.) Thus the condition is

$$v \gg \sqrt{\frac{d}{a_B}} v_e \approx 4 \times 10^3 \frac{m}{s} \quad (26)$$

In other terms, the bombarding neutron energy must be much larger than 0.08 eV. This ensures that the electron- $\alpha$  interaction is negligible during the neutron- $\alpha$  collision.

The suggested measurement is just a measurement of the differential scattering cross section of the neutron-helium collision (the neutron is detected) under the conditions described above. We also assume that the incoming neutron can be described by a plane wave in a satisfactory manner. First order time dependent perturbation theory gives a suitable approximation for the wave function of the whole system, then we calculate the probability of observing an outgoing neutron at a given angle with arbitrary momentum. Thus we get for the differential scattering cross section the formula (in laboratory frame)

$$\begin{aligned}\frac{d\sigma}{d\Omega} &= \frac{m_n g^2}{(2\pi\hbar)^3 k} \int_0^\infty dk' \int_{-\infty}^\infty d\tau \int d^3 \vec{y}_\parallel k'^2 \\ &\quad \times \exp \left[ -i\hbar \left( \frac{k^2 - k'^2}{2m_n} - \frac{(\vec{k} - \vec{k}')^2}{2M_\alpha} \right) \tau \right] \\ &\quad \times \rho \left( \vec{y}_\parallel + \frac{\hbar}{2M_\alpha} (\vec{k} - \vec{k}') \tau, \vec{y}_\parallel - \frac{\hbar}{2M_\alpha} (\vec{k} - \vec{k}') \tau \right)\end{aligned}\quad (27)$$

Here  $\vec{k}$ ,  $\vec{k}'$  stand for the wave vectors of the incoming and the scattered neutrons, respectively. The explicit

appearance of the density matrix of the  $\alpha$  particle is just a sign of decoherence, which in turn is a consequence of the negligible interaction between the nucleus and the electrons during the collision. In order to calculate the density matrix of the nucleus of the helium atom we again apply the Born-Oppenheimer approximation (6) where instead of the one-electron wave function we have a two-electron wave function. For the ground state we use the hydrogen-like wave function

$$\varphi_0(\vec{r}_1, \vec{r}_2, \sigma_1, \sigma_2) = \frac{Z^{*3}}{\pi a_B^3} \exp\left(-\frac{Z^*}{a_B}(r_1 + r_2)\right) \times \chi^0(\sigma_1, \sigma_2). \quad (28)$$

Here  $Z^* = \frac{27}{16}$  is the effective atomic number and  $\chi^0(\sigma_1, \sigma_2)$  stands for the singlet spin function. Using Eq.(28) we get for the density matrix of the nucleus

$$\rho(\vec{r}_\alpha, \vec{r}_\alpha') = \psi(\vec{r}_\alpha) \psi^*(\vec{r}_\alpha') \left(1 + s + \frac{1}{3}s^2\right)^2 e^{-2s} \quad (29)$$

where  $s = Z^*|\vec{r}_\alpha - \vec{r}_\alpha'|/a_B$ .

Inserting Eqs.(29) and (11) into Eq.(27) we get (if  $P_0 = 0$ )

$$\begin{aligned} \frac{d\sigma}{d\Omega} &= \frac{m_n g^2}{(2\pi\hbar)^3 k} \int_{-\infty}^{\infty} d\tau \int_0^{\infty} dk' k'^2 \\ &\times e^{-2\kappa|\tau|} \left(1 + \kappa|\tau| + \frac{1}{3}\kappa^2\tau^2\right)^2 \\ &\times \exp\left[-i\hbar\left(\frac{k^2 - k'^2}{2m_n} - \frac{(\vec{k} - \vec{k}')^2}{2m_\alpha}\right)\tau - \frac{1}{8}z_0^2\kappa^2\tau^2\right] \end{aligned} \quad (30)$$

where

$$\kappa = \frac{\hbar}{m_\alpha} \frac{|\vec{k} - \vec{k}'|}{a_B} Z^* \quad (31)$$

Finally, by doing the integrations in the limit  $a_B \ll \delta$  and

$$q = ka_B = \frac{\sqrt{2m_n E_n}}{\hbar} a_B \gg 1 \quad (32)$$

(note that Eq.(26) implies  $q \gg 3.4$ ) we have (by taking  $m_\alpha = 4m_n$ )

$$\frac{d\sigma}{d\Omega} = \frac{m_n^2 g^2}{25\pi^2 \hbar^4} f(\vartheta) \left(1 + \frac{h(\vartheta)}{q^2} + \mathcal{O}\left(\frac{1}{q^4}\right)\right). \quad (33)$$

where

$$f(\vartheta) = \frac{(\cos\vartheta + \sqrt{15 + \cos^2\vartheta})^2}{\sqrt{15 + \cos^2\vartheta}} \quad (34)$$

and

$$h(\vartheta) = \frac{6075}{64} \frac{3 + 5\cos^2\vartheta}{(15 + \cos^2\vartheta)^2 (\cos\vartheta + \sqrt{15 + \cos^2\vartheta})^2}. \quad (35)$$

Note that Eq.(33) is given in laboratory frame and the angular dependence (34) of the leading order corresponds just to isotropic scattering in the center of mass frame. Decoherence shows up as an anomalous contribution to the differential scattering cross section that is inversely proportional to the bombarding energy. This contribution is always positive and has maxima at  $\vartheta = 0$  and  $\vartheta = \pi$ . At  $E_n = 1$  eV these maxima are  $h(0)/q^2 = 8.2 \times 10^{-4}$  and  $h(\pi)/q^2 = 2.27 \times 10^{-3}$ .

In conclusion we add that other kinds of interactions (e.g., electron-photon or gravitational) may also result in an almost diagonal reduced density matrix. Again, one has to be careful in interpreting it as a loss of coherence. A more detailed discussion of these questions will be presented elsewhere.

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- [1] D.Giuliani, E.Joos, C.Kiefer, J.Kupsch, I.-O.Stamatescu and H.D.Zeh, *Decoherence and the Appearance of a Classical World*, (Springer, 1995) and references therein.
  - [2] M.Born, J.R.Oppenheimer, Ann.Physik **84** (1927) 457.
  - [3] C.S.Adams, M.Sigel, J.Mlynek, *Atom optics*, Phys.Rep. **240** (1994) and references therein.
  - [4] J.Franck, Trans.Faraday Soc. **21** (1925) 536; E.U.Condon, Phys.Rev. **32** (1928) 838; **41** (1932) 759.
  - [5] E.Schmidt, Math. Annalen . **63** (1907) 433; A.C.Zaanen, *Linear Analysis* (North Holland, Amsterdam, 1960) pp.432-440; O.Kübler and H.D.Zeh, Ann.Phys. (NY) **76** (1973) 405.